

The microwave *radian frequency*  $\omega_0$  is related to the *frequency*  $f_0$  in oscillations per second (Hertz) by  $\omega_0=2\pi f_0$ . For any wave, its wavelength  $\lambda_m$  in the medium and frequency  $f_0$  are related to their propagation velocity by  $f_0\lambda_m=c_m$ , where  $c_m=(\mu\epsilon)^{-1/2}$  is the phase velocity of the wave in the medium. In the important case of free space (including the Earth's atmosphere to first order), the velocity of propagation takes on its reference value, which is the speed of light,  $c=3\times 10^8$  ms<sup>-1</sup>. The free space wavelength is  $\lambda$ . It is common to denote the free space wavelength by the *wavenumber*  $k=2\pi/\lambda$ , with the corresponding definition  $k_m$  for the wavenumber within a medium.

The third constant deserves more discussion. The *complex permittivity*  $\epsilon_c$ , often called the *dielectric constant*, is the principal description of the medium's response to the presence of an electric field. Formally,

$$\epsilon_c = \epsilon - j \frac{\sigma}{\omega_0} = \epsilon' - j\epsilon'' = \epsilon_0(\epsilon' - j\epsilon'') \quad (2.34)$$

where  $\epsilon$  is the permittivity of the material,  $\epsilon_0=8.85\times 10^{-12}$  farad/m is the permittivity (dielectric constant) of free space in standard MKS units, and  $\epsilon'$  is the *dielectric constant* of the material. Yes, there is potential confusion here! Actually,  $\epsilon'$  is the *relative dielectric constant*, but that cumbersome term seldom is used. The (relative) dielectric constant is the number usually cited as an intrinsic property of a given medium. In these expressions,  $\sigma$  is the conductivity of the material (units of mhos), the inverse of its resistance (units of Ohms). Caution! Both  $\epsilon$  and  $\epsilon'$  may be found in the literature, but the associated distinction between the absolute and relative values indicated by these two fonts is not always reliable.

The lossy part of the dielectric constant is given by  $\epsilon''$  (or  $\epsilon''$ ). It is customary to express this contribution in terms of the *loss tangent*

$$\tan\delta = \frac{\epsilon''}{\epsilon'} = \frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega_0\epsilon} \quad (2.35)$$

which is tabulated in a variety of sources for many materials. In general, a small loss tangent  $\tan\delta\ll 1$  implies a low loss dielectric, and a material with large loss tangent  $\tan\delta\gg 1$  is a good conductor of electricity.

The material's dielectric constant depends weakly on frequency, but its loss tangent depends strongly on frequency. Typical values (Ramo and Whinnery 1953, p. 312) of these parameters are:

Material	$\epsilon'$	Frequency at which $\sigma=\omega_0\epsilon$
Sea water	81	$8.9 \times 10^8$
Fresh water	81	$2.2 \times 10^5$
Wet earth	10	$1.8 \times 10^6$
Dry earth	5	$3.6 \times 10^4$

Taken from Keith Rayne: Radar Fundamentals, Cap. 2 in Floyd M. Henderson, Principles & Applications of Imaging Radar